

**KUNCI UN MATEMATIKA SMA IPA
JANUARI
PAKET 3**

1. **Jawaban: D**

Pembahasan:

Diketahui : $U_1 = 80.000.000,00$ dan $r = 1/4$

Ditanya : $U_3 ?$

Pembahasan :

$$U_3 = 80.000.000 \left(\frac{3}{4}\right)^{3-1}$$

$$= 80.000.000 \left(\frac{9}{16}\right)$$

$$= 45.000.000$$

Jadi, nilai jual mobil setelah dipakai 3 tahun adalah Rp 45.000.000,00

2. **Jawaban: A**

Pembahasan:

a. Jarak titik A ke titik B = panjang garis AB = 4 cm.

b. Jarak titik A ke titik C = panjang diagonal AC = $4\sqrt{2}$ cm.

c. Jarak titik A ke titik D = panjang garis AD = 4 cm.

d. Jarak titik A ke titik G = panjang garis \overline{AG}

$$AG = \sqrt{AC^2 + CG^2} = \sqrt{(4\sqrt{2})^2 + 4^2} = \sqrt{32+16} = \sqrt{48} = 4\sqrt{3} \text{ cm}$$

e. Jarak titik A ke garis BC = panjang garis AB = 4 cm.

f. Jarak titik C ke garis FH = CO, di mana titik O adalah titik pertengahan FH.

Perhatikan $\triangle COF$, $CF = 4\sqrt{2}$ cm, $OF = 2\sqrt{2}$ cm. Maka:

$$CO = \sqrt{CF^2 - OF^2} = \sqrt{(4\sqrt{2})^2 - (2\sqrt{2})^2} = \sqrt{32-8} = \sqrt{24} = 2\sqrt{6} \text{ cm}$$

g. Jarak titik P ke garis BD adalah PR, dengan R titik di tengah garis BD.

Perhatikan $\triangle RCP$ siku-siku di C, $RC = 2\sqrt{2}$ cm, dan $PC = 2$ cm.

$$PR = \sqrt{RC^2 + PC^2} = \sqrt{(2\sqrt{2})^2 + 2^2} = \sqrt{8+4} = \sqrt{12} = 2\sqrt{3} \text{ cm}$$

3. **Jawaban: A**

Pembahasan:

$$\tan \frac{1}{2}x = t \Rightarrow \sin \frac{1}{2}x = \frac{t}{\sqrt{t^2+1}} \text{ dan } \cos \frac{1}{2}x = \frac{1}{\sqrt{t^2+1}}$$

$$\sin x = 2 \sin \frac{1}{2}x \cos \frac{1}{2}x = 2 \cdot \frac{t}{\sqrt{t^2+1}} \cdot \frac{1}{\sqrt{t^2+1}} = \frac{2t}{t^2+1}$$

4. **Jawaban: B**

Pembahasan:

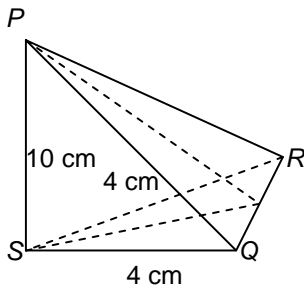
$$\cos(\alpha - \beta) = \frac{1}{2}\sqrt{3}$$

$$\cos\alpha\cos\beta + \sin\alpha\sin\beta = \frac{1}{2} + \sin\alpha\sin\beta = \frac{1}{2}\sqrt{3} \Leftrightarrow \sin\alpha\sin\beta = \frac{1}{2}\sqrt{3} - \frac{1}{2}$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta = \frac{1}{2} - \left(\frac{1}{2}\sqrt{3} - \frac{1}{2}\right) = 1 - \frac{1}{2}\sqrt{3}$$

$$\frac{\cos(\alpha + \beta)}{\cos(\alpha - \beta)} = \frac{1 - \frac{1}{2}\sqrt{3}}{\frac{1}{2}\sqrt{3}} = \frac{2}{3}\sqrt{3} - 1$$

5. **Jawaban: A**
Pembahasan:



\Leftrightarrow PS tegak lurus alas, maka:

PS tegak lurus PR dan PS tegak lurus SQ

$\Leftrightarrow \triangle QSR$ siku-siku di Q, maka:

$$\begin{aligned} ST &= SQ \cos \frac{90^\circ}{2} \\ &= SQ \cos 45^\circ \\ &= 4 \left(\frac{1}{2}\sqrt{2} \right) = 2\sqrt{2} \text{ cm} \end{aligned}$$

$$\begin{aligned} \Leftrightarrow \tan \alpha &= \frac{PS}{ST} = \frac{10}{2\sqrt{2}} = \frac{5}{2}\sqrt{2} \\ &= \frac{5}{2}\sqrt{2} \text{ cm} \end{aligned}$$

6. **Jawaban: A**
Pembahasan:

\Leftrightarrow HF = AF = AH = diagonal sisi

panjangnya $14\sqrt{2}$ cm

$$HT = \frac{1}{2}HF = 7\sqrt{2} \text{ cm}$$

\Leftrightarrow Panjang proyeksi AH pada ACGE adalah At:

$$\begin{aligned} AT &= \sqrt{AH^2 - HT^2} = \sqrt{(14\sqrt{2})^2 - (7\sqrt{2})^2} \\ &= \sqrt{392 - 98} = \sqrt{294} \\ &= 7\sqrt{6} \text{ cm} \end{aligned}$$

7. **Jawaban: B**
Pembahasan:

$$\tan^2 x = 1 + \sec x \Leftrightarrow \frac{\sin^2 x}{\cos^2 x} = 1 + \frac{1}{\cos x} \quad | \cdot \cos^2 x |$$

$$2 \cos^2 x + \cos x - 1 = 0 \Leftrightarrow (2 \cos x - 1)(\cos x + 1) = 0$$

$$\cos x = \frac{1}{2} \Rightarrow x = 60^\circ$$

$$\cos x = -1 \Rightarrow x = 180^\circ \text{ (tidak memenuhi karena } 0^\circ < x < 90^\circ \text{)}$$

8. **Jawaban: A**

Pembahasan:

$$\sin x = \frac{\sqrt{5}}{5} \Rightarrow \cos x = \frac{2\sqrt{5}}{5}$$

$$\cos x - 5 \cos\left(\frac{\pi}{2} + x\right) + 2 \sin(\pi - x) = \cos x - 5(-\sin x) + 2 \sin x$$

$$= \cos x + 7 \sin x = \frac{2\sqrt{5}}{5} + 7 \frac{\sqrt{5}}{5} = \frac{9}{5} \sqrt{5}$$

9. **Jawaban: C**

Pembahasan:

$$\lim_{x \rightarrow 0} \frac{1 - \sqrt{x+1}}{x^2 - x} = \lim_{x \rightarrow 0} \frac{1 - \sqrt{x+1}}{x^2 - x} \left[\frac{1 + \sqrt{x+1}}{1 + \sqrt{x+1}} \right] = \lim_{x \rightarrow 0} \frac{1 - (x+1)}{x(x-1)(1 + \sqrt{x+1})}$$

$$= \lim_{x \rightarrow 0} \frac{-1}{(x-1)(1 + \sqrt{x+1})} = \frac{1}{2}$$

10. **Jawaban: A**

Pembahasan:

$$\lim_{x \rightarrow 0} \frac{\operatorname{tg} x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{\operatorname{tg} x}{x} = \lim_{x \rightarrow 0} \frac{\sin x}{\cos x} \text{ maka } \lim_{x \rightarrow 0} = \frac{\sin x}{x} \cdot \lim_{x \rightarrow 0} \frac{1}{\cos x} = 1 \cdot 1 = 1$$