

**UJI KOMPETENSI SEMESTER 1
MATEMATIKA XII**

A. Pilihan Ganda

1. **Jawaban: c.** $\frac{13}{2}$

Pembahasan:

$$\int_0^1 (3x^2 - 3x + 7) dx = x^3 - \frac{3}{2}x^2 + 7x \Big|_0^1 = \left\{ (1)^3 - \frac{3}{2}(1)^2 + 7 \cdot 1 \right\} - 0 = 1 - \frac{3}{2} + 7 = \frac{16-3}{2} = \frac{13}{2}$$

2. **Jawaban: c.** $20\frac{5}{6}$

Pembahasan: Kurva $y = x^2$ dan garis $x + y = 6$ ($y = 6 - x$).

Substitusikan nilai y pada $y = x^2$ sehingga didapat : $6 - x = x^2$

$$6 - x = x^2$$

$$x^2 + x - 6 = 0 \quad (a = 1, b = 1, c = -6)$$

Untuk mencari luas pada soal diatas lebih mudah jika dikerjakan menggunakan rumus luas yang

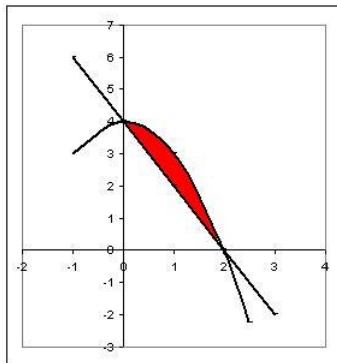
menggunakan bantuan diskriminan. $L = \frac{D\sqrt{D}}{6a^2}$.

$$D = b^2 - 4ac = 1^2 - 4(1)(-6) = 1 + 24 = 25$$

$$L = \frac{D\sqrt{D}}{6a^2} = \frac{25\sqrt{25}}{6 \cdot 1^2} = \frac{25 \cdot (5)}{6} = \frac{125}{6} = 20\frac{5}{6}$$

3. **Jawaban: d.** $\frac{8}{3}\pi$

Pembahasan:



Cat: Gambar diatas kemudian diputar 360^0 terhadap sumbu y (kasih masukkan ya, kalau anda tahu cara menggambar kurva dengan putaran 360^0).

Dari gambar sebenarnya terlihat titik potong kedua kurva. Kalau melalui perhitungan didapat dari :

$$y = -x^2 + 4$$

$$y = -2x + 4$$

Substitusikan nilai y , didapat :

$$-2x + 4 + x^2 - 4 = 0$$

$$x^2 - 2x = 0$$

$$x(x - 2) = 0$$

$$x = 0 \text{ atau } x = 2$$

Untuk nilai y , substitusikan nilai x pada $y = -2x + 4$

$$x = 0 \quad y = -2(0) + 4 = 4$$

$$x = 2 \quad y = -2(2) + 4 = 0$$

Karena bida diputar terhadap sumbu y, maka terlebih dahulu rubah fungsi $y = f(x)$ menjadi $x = f(y)$.

$$y = -x^2 + 4$$

$$y = -2x + 4$$

$$y - 4 = -x^2$$

$$y - 4 = -2x$$

$$4 - y = x^2$$

$$2 - \frac{1}{2}y = x$$

$$x = \sqrt{4 - y}$$

$$V = \pi \int_a^b f^2(y) - g^2(y) \, dy$$

$$= \pi \int_0^4 (\sqrt{4 - y})^2 - (2 - \frac{1}{2}y)^2 \, dy$$

$$= \pi \int_0^4 (4 - y) - (4 - 2y + \frac{1}{4}y^2) \, dy$$

$$= \pi \int_0^4 -\frac{1}{4}y^2 + y \, dy = -\frac{1}{12}y^3 + \frac{1}{2}y^2 \Big|_0^4 \pi$$

$$= \left\{ -\frac{1}{12}(4)^3 + \frac{1}{2}(4)^2 \right\} \pi = \left(-\frac{16}{3} + 8 \right) \pi = \frac{8}{3} \pi$$

4. **Jawaban: d.** $14 \frac{2}{5} \pi$

Pembahasan:

$$y = x^2 \text{ dan } x + y - 2 = 0 \text{ (} y = 2 - x \text{)}$$

Substitusi kedua persamaan untuk mendapat titik potongnya.

$$x^2 = 2 - x$$

$$x^2 + x - 2 = 0$$

$$(x + 2)(x - 1) = 0$$

$$x = -2 \text{ atau } x = 1$$

$$V = \pi \int_a^b f^2(x) - g^2(x) \, dx$$

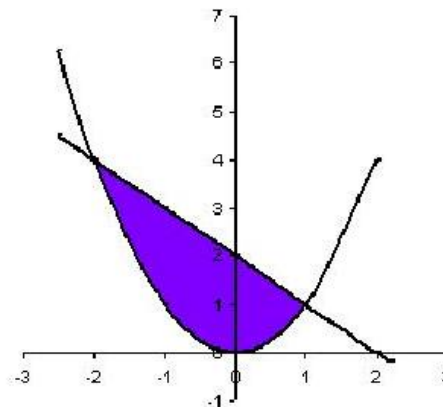
$$= \pi \int_{-2}^1 (2 - x)^2 - (x^2)^2 \, dx$$

$$= \pi \int_{-2}^1 4 - 4x + x^2 - x^4 \, dx$$

$$= \pi \left(4x - 2x^2 + \frac{1}{3}x^3 - \frac{1}{5}x^5 \right) \Big|_{-2}^1$$

$$= \pi \left\{ (4(1) - 2(1)^2 + \frac{1}{3}(1)^3 - \frac{1}{5}(1)^5) - (4(-2) - 2(-2)^2 + \frac{1}{3}(-2)^3 - \frac{1}{5}(-2)^5) \right\}$$

$$= \pi \left\{ (4 - 2 + \frac{1}{3} - \frac{1}{5}) - (-8 - 8 - \frac{8}{3} + \frac{32}{5}) \right\}$$



$$\begin{aligned}
&= \pi\left(2 + \frac{1}{3} - \frac{1}{5} + 16 + \frac{8}{3} - \frac{32}{5}\right) \\
&= \left(21 - 6\frac{3}{5}\right)\pi \\
&= 14\frac{2}{5}\pi
\end{aligned}$$

5. **Jawaban: b. 15**

Penyelesaian:

$$\frac{df(x)}{dx} = x^2 + x$$

$$\int df(x) = \int x^2 + x \, dx$$

$$f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 + c$$

$$f(2) = \frac{1}{3} \cdot 8 + \frac{1}{2} \cdot 4 + c = \frac{20}{3}$$

$$\frac{16}{6} + \frac{12}{6} + c = \frac{40}{6} \text{ sehingga } c = 2$$

$$\int_1^3 f(x) \, dx = \int_1^3 \left(\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2\right) \, dx$$

$$\begin{aligned}
&= \left. \frac{1}{12}x^4 + \frac{1}{6}x^3 + 2x \right|_1^3 \\
&= \left(\frac{81}{12} + \frac{54}{12} + \frac{72}{12}\right) - \left(\frac{1}{12} + \frac{2}{12} + \frac{24}{12}\right) \\
&= \frac{180}{12} = \frac{30}{2} = 15
\end{aligned}$$

6. **Jawaban: e.** $\frac{1}{11}p^{11} + C$

Pembahasan:

$$\int (3 + \sin x)^{10} \cos x \, dx$$

andai $p = 3 + \sin x$ maka $dp = \cos x \, dx$

$$\int p^{10} dp = \frac{1}{11}p^{11} + c$$

7. **Jawaban: a.** $\frac{1}{2}(x+5)\sin 2x + \frac{1}{4}\cos 2x + C$

Penyelesaian :

$$\int (x+5) \cos 2x \, dx$$

Misal :

$$u = x + 5 \Rightarrow du = dx$$

$$dv = \cos 2x \, dx \Rightarrow v = \frac{1}{2}\sin 2x$$

$$\int u \, dv = uv - \int v \, du$$

$$\int (x+5)\cos 2x = (x+5) \cdot \frac{1}{2}\sin 2x - \int \frac{1}{2}\sin 2x dx = \frac{1}{2}(x+5)\sin 2x - \frac{1}{2}\left(-\frac{1}{2}\cos 2x\right) + C$$

$$= \frac{1}{2}(x+5)\sin 2x + \frac{1}{4}\cos 2x + C$$

8. **Jawaban: b.** $21\frac{1}{3}$

Pembahasan:

$$f(x) = (x-2)^2 - 4$$

$$= x^2 - 4x + 4 - 4$$

$$= x^2 - 4x \quad (\text{terbuka keatas})$$

$$-f(x) = 4x - x^2 \quad (\text{terbuka kebawah})$$

Note : Untuk mengetahui bentuk sebuah kurva dapat dilihat pada koefisien x^2 , jika positif maka kurva terbuka keatas, dan jika negatif terbuka kebawah.

Batas atas dan bawah didapat dari akar – akar $x^2 - 4x$.

$$x^2 - 4x = 0$$

$$x(x-4) = 0$$

$$x = 0 \quad \text{atau} \quad x - 4 = 0$$

$$x = 0 \quad \text{atau} \quad x = 4$$

$$L = \int_a^b f(x) - g(x) \, dx$$

$$= \int_0^4 (4x - x^2) - (x^2 - 4x) \, dx$$

$$= \int_0^4 4x - x^2 - x^2 + 4x \, dx$$

$$= \int_0^4 8x - 2x^2 \, dx$$

$$= 4x^2 - \frac{2}{3}x^3 \Big|_0^4 = \{4(4)^2 - \frac{2}{3}(4)^3\} - \{4(0)^2 - \frac{2}{3}(0)^3\}$$

$$= 64 - \frac{128}{3} = 64 - \frac{128}{3} = 21\frac{1}{3}$$

9. **Jawaban: e.** $\int_2^4 (x-2)dx + \int_4^5 ((x-2) - (x^2 - 6x + 8))dx$

Pembahasan:

$$\int_2^4 (x-2)dx + \int_4^5 ((x-2) - (x^2 - 6x + 8))dx$$

10. **Jawaban: b.** $\frac{83}{15}\pi$

Pembahasan:

$$\begin{aligned}
V &= \pi \int_{-1}^0 (-4x)^2 - (x^2)^2 dx \\
&= \pi \int_{-1}^0 16x^2 - 4^4 dx \\
&= \pi \left[\frac{16x^3}{3} - \frac{x^5}{5} \right]_{-1}^0 \\
&= \pi \left(\left(\frac{16}{3}(0)^3 - \frac{1}{5}(0)^2 \right) - \left(\frac{16}{3}(-1)^3 - \frac{1}{5}(-1)^2 \right) \right) \\
&= \pi \left(0 - \left(-\frac{16}{3} - \frac{1}{5} \right) \right) \\
&= \pi \left(\frac{16}{3} + \frac{1}{5} \right) = \frac{83}{15} \pi
\end{aligned}$$

11. **Jawaban: c. -1**

Pembahasan:

$$\int_p^3 (3x^2 - 2x + 2) dx = x^3 - x^2 + 2x \Big|_p^3 = 40.$$

$$x^3 - x^2 + 2x \Big|_p^3 = \{3^3 - 3^2 + 2(3)\} - \{p^3 - p^2 + 2p\} = 40$$

$$27 - 9 + 6 - p^3 + p^2 - 2p = 40$$

$$24 - p^3 + p^2 - 2p - 40 = 0$$

$$-p^3 + p^2 - 2p - 16 = 0 \quad (\text{kalikan kedua ruas dengan } (-))$$

$$p^3 - p^2 + 2p + 16 = 0 \quad (\text{gunakan suku banyak untuk mendapatkan nilai } p)$$

Untuk menentukan nilai p dapat dicari dengan menentukan faktor dari perkalian koefisien p^3 dan p^0 yaitu 1 dan

16. Faktor – faktor yang mungkin adalah : $\pm 16, \pm 8, \pm 4, \pm 2, \pm 1$. Karena nilai a yang memenuhi adalah -2 maka nilai $\frac{1}{2} p = -1$

12. **Pembahasan: d.** $\begin{bmatrix} -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{bmatrix}$

$$D^{-1} = \frac{1}{-3} \begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{bmatrix}$$

13. **Pembahasan: a.** 800

$$\begin{aligned}
\det A &= 20 \cdot 40 - 0 \cdot 4 \\
&= 800
\end{aligned}$$

14. **Pembahasan: a.** 250

$$\det A = 25 \cdot 150 - 40 \cdot 100$$

$$= 3.750 - 4.000$$

$$= 250$$

15. **Jawaban: a.** $\begin{pmatrix} 1 \\ 6 \end{pmatrix}$

Pembahasan:

$$\overline{AB} = \overline{OB} - \overline{OA}$$

$$= \begin{pmatrix} 5 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ -3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 6 \end{pmatrix}$$

16. **Jawaban: a.** $\bar{j} + \bar{k}$

Pembahasan:

$$\overline{AB} = \bar{b} - \bar{a} = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} - \begin{bmatrix} 6 \\ 6 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$$

$$\overline{AC} = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

$$\bar{i} = \frac{(\overline{AB} \cdot \overline{AC})}{|\overline{AC}|^2} \cdot \overline{AC}$$

$$\bar{i} = \frac{\begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}}{4+4} \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

$$\bar{i} = \frac{1}{2} \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \bar{j} + \bar{k}$$

17. **Jawaban: d.** 90°

Pembahasan:

$$RP = p - r = \begin{bmatrix} 0 \\ 1 \\ 4 \end{bmatrix} - \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$RQ = q - r = \begin{bmatrix} 2 \\ -3 \\ 2 \end{bmatrix} - \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ 0 \end{bmatrix}$$

$$PR \cdot RQ = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ -3 \\ 0 \end{bmatrix} = 3 - 3 + 0 = 0$$

$$\cos \angle PRQ = \frac{\overline{PR} \cdot \overline{RQ}}{|PR| \cdot |RQ|} = 0$$

$$\angle PRQ = 90^\circ$$

18. **Jawaban: d. 135°**

Pembahasan:

$$a \cdot b = 1 \cdot (-1) + (-1) \cdot 2 + 0 \cdot 2 = -1 - 2 + 0 = -3$$

$$|a| = \sqrt{(1)^2 + (-1)^2 + (0)^2} = \sqrt{2}$$

$$|b| = \sqrt{(-1)^2 + (2)^2 + (2)^2} = 3$$

Misalkan sudut yang dibentuk oleh vektor a dan vektor b adalah α , maka:

$$\cos \alpha = \frac{a \cdot b}{|a||b|} = \frac{-3}{\sqrt{2} \cdot 3} = -\frac{1}{\sqrt{2}} = -\frac{1}{2}\sqrt{2}. \text{ Didapat } \alpha = 135^\circ.$$

19. **Jawaban: e. $\frac{1}{2}$**

Pembahasan:

$$a = (2, -1, 2) \text{ dan } b = (-1, 1, 2)$$

$$a \cdot b = 2 \cdot (-1) + (-1) \cdot 1 + 2 \cdot 2 = -2 - 1 + 4 = 1$$

$$|a| = \sqrt{(2)^2 + (-1)^2 + (2)^2} = \sqrt{4+1+4} = 3$$

$$|b| = \sqrt{(-1)^2 + (1)^2 + (2)^2} = \sqrt{1+1+4} = 2$$

Misalkan panjang proyeksi vektor a pada b adalah c, maka:

$$|c| = \frac{a \cdot b}{|b|} = \frac{1}{2}$$

20. **Jawaban: c. $\frac{2}{3}i - \frac{1}{3}j + \frac{2}{3}k$**

Pembahasan:

$$a = 2i - j + 2k \text{ dan } b = -i + j + 2k$$

$$a \cdot b = 2 \cdot (-1) + (-1) \cdot 1 + 2 \cdot 2 = -2 - 1 + 4 = 1$$

$$|a| = \sqrt{(2)^2 + (-1)^2 + (2)^2} = \sqrt{4+1+4} = 3$$

$$|b| = \sqrt{(-1)^2 + (1)^2 + (2)^2} = \sqrt{1+1+4} = 2$$

Misalkan panjang proyeksi vektor a pada b adalah c, maka:

$$|c| = \frac{a \cdot b}{|a|^2} \cdot a = \frac{1}{3} \cdot (2i - j + 2k) = \frac{2}{3}i - \frac{1}{3}j + \frac{2}{3}k$$

21. **Jawaban: b. -3**

Pembahasan:

$$6 = \frac{-2x - 4 + 2y}{\sqrt{4+1+4}} \rightarrow 18 = -2x - 4 + 2y$$

$$22 = -2x + 2y$$

$$11 = -x + y \rightarrow y = x + 11$$

$$|\vec{a}| = \sqrt{89}$$

$$\sqrt{89} = \sqrt{x^2 + 16 + y^2}$$

$$\sqrt{89} = \sqrt{x^2 + 16 + y^2 + (11+x)^2}$$

$$89 = x^2 + 16 + 121 + 22x + x^2$$

$$2x^2 + 22x + 48 = 0 \rightarrow x^2 + 11x + 24 = 0$$

$$(x+3)(x+8) = 0$$

$$x = -3 \text{ atau } x = -8$$

22. Jawaban: b. 9

Pembahasan:

$$\vec{a} \cdot \vec{b} = a_1 \cdot b_1 + a_2 \cdot b_2 + a_3 \cdot b_3 = 0$$

$$= 1 \cdot 2 + 2 \cdot (-10) + m \cdot 2 = 0$$

$$2 - 20 + 2m = 0$$

$$-18 + 2m = 0$$

$$2m = 18$$

$$m = 9$$

23. Jawaban: e. 60°

Pembahasan:

$$\cos \alpha = \frac{a_1 b_1 + a_2 b_2 + a_3 b_3}{\sqrt{(a_1)^2 + (a_2)^2 + (a_3)^2} \times \sqrt{(b_1)^2 + (b_2)^2 + (b_3)^2}}$$

$$\cos \alpha = \frac{2 \cdot (-1) + 1 \cdot 3 + (-3) \cdot (-2)}{\sqrt{(2)^2 + (1)^2 + (-3)^2} \times \sqrt{(-1)^2 + (3)^2 + (-2)^2}}$$

$$\cos \alpha = \frac{-2 + 3 + 6}{\sqrt{4 + 1 + 9} \times \sqrt{1 + 9 + 4}}$$

$$\cos \alpha = \frac{7}{\sqrt{14} \times \sqrt{14}}$$

$$\cos \alpha = \frac{7}{14}$$

$$\cos \alpha = \frac{1}{2}, \text{ maka } \alpha = 60^\circ \text{ karena } \cos \alpha = \frac{1}{2}$$

24. Jawaban: c. $\frac{1}{3}$

Pembahasan:

$$\begin{aligned} \left| \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} \right| &= \frac{\begin{bmatrix} -\sqrt{3} \\ p \\ 1 \end{bmatrix} \cdot \begin{bmatrix} -\sqrt{3} \\ 2 \\ p \end{bmatrix}}{\sqrt{(-\sqrt{3})^2 + 2^2 + p^2}} = \frac{3 + 2p + p}{\sqrt{3 + 4 + p^2}} = \frac{3 + 3p}{\sqrt{7 + p^2}} = \frac{3}{2} \\ \Leftrightarrow 2(3 + 3p) &= 3\sqrt{7 + p^2} \\ \Leftrightarrow 6(1 + p) &= 3\sqrt{7 + p^2} \\ \Leftrightarrow 2(1 + p) &= \sqrt{7 + p^2} \\ \Leftrightarrow (2 + 2p)^2 &= (\sqrt{7 + p^2})^2 \\ \Leftrightarrow 4 + 8p + 4p^2 &= 7 + p^2 \\ \Leftrightarrow 8p + 3p^2 - 3 &= 0 \\ \Leftrightarrow (3p - 1)(p + 3) &= 0 \\ \Leftrightarrow p &= \frac{1}{3} \\ \Leftrightarrow p &= -3(\text{tidak}) \end{aligned}$$

25. **Jawaban: a. $3x + y + 2 = 0$**

Pembahasan:

Misalnya (a,b) pada kurva $y - 3x - 2 = 0$

$$\begin{bmatrix} a \\ b \end{bmatrix} = \frac{1}{-1-0} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a' \\ b' \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \end{bmatrix} = -1 \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a' \\ b' \end{bmatrix}$$

$$a = a'$$

$$b = -b'$$

Sehingga :

$$b - 3a - 2 = 0$$

$$-b' - 3a' - 2 = 0$$

$$-y' - 3x' - 2 = 0$$

$$\begin{bmatrix} x'' \\ y'' \end{bmatrix} = \begin{bmatrix} \cos 90^\circ & -\sin 90^\circ \\ \sin 90^\circ & \cos 90^\circ \end{bmatrix} \begin{bmatrix} a \\ -b \end{bmatrix}$$

$$\begin{bmatrix} x'' \\ y'' \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a' \\ -b' \end{bmatrix}$$

$$\begin{bmatrix} x'' \\ y'' \end{bmatrix} = \begin{bmatrix} b' \\ a' \end{bmatrix}$$

$$-x'' - 3y'' - 2 = 0$$

$$x'' + 3y'' + 2 = 0$$

B. Uraian

1. Pembahasan:

Mencari batas-batas luas daerah:

$$y = x^2 \quad \text{dan} \quad x + y = 6 \quad y = 6 - x$$

$$\begin{aligned} \text{substitusi } y = x^2 \text{ ke } y = 6 - x &\rightarrow x^2 = 6 - x \\ &\rightarrow x^2 + x - 6 = 0 \\ &\rightarrow (x + 3)(x - 2) = 0 \\ &\rightarrow x = -3 \text{ dan } x = 2 \end{aligned}$$

$$L = \frac{D\sqrt{d}}{6a^2} \rightarrow L = \frac{(b^2 - 4ac)(\sqrt{b^2 - 4ac})}{6a^2} = \frac{(1^2 - 4 \cdot 1(-6))\sqrt{1^2 - 4(1)(-6)}}{6(1)^2} = 20\frac{5}{6} \text{ satuan luas}$$

2. **Penyelesaian:**

Stasiun 1 : 480 menit – 48 menit = 432 menit

Stasiun 2 : 480 menit – 67.2 menit = 412.8 menit

Stasiun 3 : 480 menit – 57.6 menit = 422.4 menit.

Model umum pemrograman linier :

Maksimumkan $z = x_1 + x_2$

Kendala :

$$6x_1 + 4x_2 \leq 432$$

$$5x_1 + 5x_2 \leq 412.8$$

$$4x_1 + 6x_2 \leq 422.4$$

$$x_1, x_2 \geq 0$$

3. **Penyelesaian:**

$$\begin{aligned} \Rightarrow \Delta &= \begin{vmatrix} 2 & 3 \\ 1 & -4 \end{vmatrix} = -11 \\ \text{a. } \left. \begin{array}{l} 2x + 3y = 2 \\ x - 4y = 1 \end{array} \right\} &\Rightarrow \Delta_x = \begin{vmatrix} 2 & 3 \\ 1 & -4 \end{vmatrix} = -11 \\ &\Rightarrow \Delta_y = \begin{vmatrix} 2 & 2 \\ 1 & 1 \end{vmatrix} = 0 \\ &\Rightarrow x = \frac{-11}{-11} = 1, y = \frac{0}{-11} = 0 \end{aligned} \quad \text{Jadi, Hpnya} = \{1,0\}$$

$$\begin{aligned} \Rightarrow \Delta &= \begin{vmatrix} 2 & 3 \\ 1 & 5 \end{vmatrix} = 12 \\ \text{b. } \left. \begin{array}{l} 2x + 3y = 5 \\ x + 5y = 6 \end{array} \right\} &\Rightarrow \Delta_x = \begin{vmatrix} 5 & 3 \\ 6 & 5 \end{vmatrix} = 7 \\ &\Rightarrow \Delta_y = \begin{vmatrix} 2 & 5 \\ 1 & 6 \end{vmatrix} = 7 \\ &\Rightarrow x = \frac{7}{7} = 1, y = \frac{7}{7} = 1 \end{aligned} \quad \text{Jadi, Hpnya} = \{1,1\}$$

4. **Penyelesaian:**

a. Komponen vektor $\overline{AB} = \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \end{pmatrix} = \begin{pmatrix} -4 - 2 \\ 5 - (-3) \end{pmatrix} = \begin{pmatrix} -6 \\ 8 \end{pmatrix}$

b. Besar vektor $\overline{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
 $= \sqrt{(-6)^2 + (8)^2} = \sqrt{36 + 64} = \sqrt{100} = 10$

5. **Penyelesaian:**

$$\overrightarrow{AB} \cdot \overrightarrow{AC} = |\overrightarrow{AB}| \cdot |\overrightarrow{AC}| \cdot \cos \alpha$$

$$\overrightarrow{AB} = \vec{b} - \vec{a} = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} - \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$$

$$\overrightarrow{AC} = \vec{c} - \vec{a} = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix} - \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

$$\cos \alpha = \frac{\overrightarrow{AB} \cdot \overrightarrow{AC}}{|\overrightarrow{AB}| |\overrightarrow{AC}|} = \frac{\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}}{\sqrt{(-1)^2 + 1^2 + (-1)^2} \cdot \sqrt{0^2 + 1^2 + (-1)^2}}$$

$$\cos \alpha = \frac{0+1+1}{\sqrt{3}\sqrt{2}} = \frac{2}{\sqrt{6}} = \frac{1}{3}\sqrt{6}$$

$$\text{Jadi, } \alpha = \left(\frac{1}{2} \sqrt{2}, \frac{1}{2} \sqrt{2} \right)$$