

PEMBAHASAN TRIGONOMETRI

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1. Jawab: A

$$\begin{aligned}\cos 105 &= \cos(60 + 45) \\ \cos 105 &= \cos 60 \cdot \cos 45 - \sin 60 \cdot \sin 45 \\ \cos 105 &= \frac{1}{2} \cdot \frac{1}{2} \sqrt{2} - \frac{1}{2} \sqrt{3} \cdot \frac{1}{2} \sqrt{2} \\ \cos 105 &= \frac{1}{4} \sqrt{2} (1 - \sqrt{3})\end{aligned}$$

2. Jawab: D

$$\begin{aligned}\cos 75 &= \cos(30 + 45) \\ \cos 75 &= \cos 30 \cdot \cos 45 - \sin 30 \cdot \sin 45 \\ \cos 75 &= \frac{1}{2} \sqrt{3} \cdot \frac{1}{2} \sqrt{2} - \frac{1}{2} \cdot \frac{1}{2} \sqrt{2} \\ \cos 75 &= \frac{1}{4} \sqrt{6} - \frac{1}{4} \sqrt{2}\end{aligned}$$

3. Jawab: A

$$\begin{aligned}\sin \alpha = \frac{4}{5} \text{ maka } \cos \alpha &= \frac{3}{5} \text{ dan karena } \sin \beta = \frac{12}{13} \text{ maka } \cos \beta = \frac{5}{13}. \text{ Jadi} \\ \cos(\alpha + \beta) &= \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta = \frac{3}{5} \times \frac{5}{13} - \frac{4}{5} \times \frac{12}{13} = \frac{15}{65} - \frac{48}{65} = -\frac{33}{65}\end{aligned}$$

4. Jawab: A

$$\begin{aligned}\cos 63 \cdot \cos 87 - \sin 63 \cdot \sin 87 &= \cos(63 + 87) = \cos 150 = \cos(90 + 60) = -\sin 60 \\ &= -\frac{1}{2} \sqrt{3}\end{aligned}$$

5. Jawab: B

$$\begin{aligned}\sin 105 &= \sin(60 + 45) \\ \sin 105 &= \sin 60 \cdot \cos 45 + \cos 60 \cdot \sin 45 \\ \sin 105 &= \frac{1}{2} \sqrt{3} \cdot \frac{1}{2} \sqrt{2} + \frac{1}{2} \cdot \frac{1}{2} \sqrt{2} \\ \sin 105 &= \frac{1}{4} \sqrt{2} (1 + \sqrt{3})\end{aligned}$$

6. Jawab: E

$$\begin{aligned}\sin 75 &= \sin(30 + 45) \\ \sin 75 &= \sin 30 \cdot \cos 45 + \cos 30 \cdot \sin 45 \\ \sin 75 &= \frac{1}{2} \cdot \frac{1}{2} \sqrt{2} + \frac{1}{2} \sqrt{3} \cdot \frac{1}{2} \sqrt{2} \\ \sin 75 &= \frac{1}{4} \sqrt{6} + \frac{1}{4} \sqrt{2}\end{aligned}$$

7. Jawab: D

$$\begin{aligned}\sin \alpha = \frac{4}{5} \text{ maka } \cos \alpha &= \frac{3}{5} \text{ dan karena } \sin \beta = \frac{12}{13} \text{ maka } \cos \beta = \frac{5}{13}. \text{ Jadi} \\ \sin(\alpha + \beta) &= \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta = \frac{4}{5} \times \frac{5}{13} + \frac{3}{5} \times \frac{12}{13} = \frac{20}{65} + \frac{36}{65} = \frac{56}{65}\end{aligned}$$

8. Jawab: D

$$\sin 63 \cdot \cos 87 + \cos 63 \cdot \sin 87 = \sin(63 + 87) = \sin 150 = \sin(90 + 60) = \cos 60 = \frac{1}{2}$$

9. Jawab: B

$$\begin{aligned}\cos 15 &= \cos(60 - 45) \\ \cos 15 &= \cos 60 \cdot \cos 45 + \sin 60 \cdot \sin 45 \\ \cos 15 &= \frac{1}{2} \cdot \frac{1}{2} \sqrt{2} + \frac{1}{2} \sqrt{3} \cdot \frac{1}{2} \sqrt{2} \\ \cos 15 &= \frac{1}{4} \sqrt{2} (1 + \sqrt{3})\end{aligned}$$

10. Jawab: E

$$\begin{aligned}\cos(-15) &= \cos(30 - 45) \\ \cos(-15) &= \cos 30 \cdot \cos 45 + \sin 30 \cdot \sin 45 \\ \cos(-15) &= \frac{1}{2} \sqrt{3} \cdot \frac{1}{2} \sqrt{2} + \frac{1}{2} \cdot \frac{1}{2} \sqrt{2} \\ \cos(-15) &= \frac{1}{4} \sqrt{6} + \frac{1}{4} \sqrt{2}\end{aligned}$$

11. Jawab: D

$\sin \alpha = \frac{4}{5}$ maka $\cos \alpha = \frac{3}{5}$ dan karena $\sin \beta = \frac{12}{13}$ maka $\cos \beta = \frac{5}{13}$. Jadi

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta = \frac{3}{5} \times \frac{5}{13} + \frac{4}{5} \times \frac{12}{13} = \frac{15}{65} + \frac{48}{65} = \frac{63}{65}$$

12. Jawab: E

$$\cos 70 \cdot \cos 25 + \sin 70 \cdot \sin 25 = \cos(70 - 25) = \cos 45 = \frac{1}{2} \sqrt{2}$$

13. Jawab: C

$$\begin{aligned}\sin 15 &= \sin(60 - 45) \\ \sin 15 &= \sin 60 \cdot \cos 45 - \cos 60 \cdot \sin 45 \\ \sin 15 &= \frac{1}{2} \sqrt{3} \cdot \frac{1}{2} \sqrt{2} - \frac{1}{2} \cdot \frac{1}{2} \sqrt{2} \\ \sin 15 &= \frac{1}{4} \sqrt{2} (\sqrt{3} - 1)\end{aligned}$$

14. Jawab: D

$$\begin{aligned}\sin(-15) &= \sin(30 - 45) \\ \sin(-15) &= \sin 30 \cdot \cos 45 - \cos 30 \cdot \sin 45 \\ \sin(-15) &= \frac{1}{2} \cdot \frac{1}{2} \sqrt{2} - \frac{1}{2} \sqrt{3} \cdot \frac{1}{2} \sqrt{2} \\ \sin(-15) &= \frac{1}{4} \sqrt{2} \left(\frac{1}{2} - \sqrt{3} \right)\end{aligned}$$

15. Jawab: B

$\sin \alpha = \frac{4}{5}$ maka $\cos \alpha = \frac{3}{5}$ dan karena $\sin \beta = \frac{12}{13}$ maka $\cos \beta = \frac{5}{13}$. Jadi

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta = \frac{4}{5} \times \frac{5}{13} - \frac{3}{5} \times \frac{12}{13} = \frac{20}{65} - \frac{36}{65} = -\frac{16}{65}$$

16. Jawab: B

$$\begin{aligned} \tan 105 &= \frac{\tan 60 + \tan 45}{1 - \tan 60 \cdot \tan 45} \\ \tan 105 &= \frac{\sqrt{3} + 1}{1 - \sqrt{3} \cdot 1} \\ \tan 105 &= \frac{\sqrt{3} + 1}{1 - \sqrt{3}} = \frac{1 + \sqrt{3}}{1 - \sqrt{3}} \times \frac{1 + \sqrt{3}}{1 + \sqrt{3}} = \frac{1 + 3 + 2\sqrt{3}}{1 - 3} = \frac{4 + 2\sqrt{3}}{-2} = -(2 + \sqrt{3}) \end{aligned}$$

17. Jawab: B

$$\sin \alpha = \frac{4}{5} \text{ maka } \cos \alpha = \frac{3}{5} \text{ dan } \tan \alpha = \frac{4}{3} \text{ dan karena } \sin \beta = \frac{12}{13} \text{ maka } \cos \beta = \frac{5}{13} \text{ dan } \tan \beta = \frac{12}{5}.$$

Jadi

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \cdot \tan \beta} = \frac{\frac{4}{3} + \frac{12}{5}}{1 - \frac{4}{3} \cdot \frac{12}{5}} = \frac{\frac{(20 + 36)}{15}}{\frac{(15 - 48)}{15}} = -\frac{56}{33}$$

18. Jawab: D

$$\begin{aligned} \tan 15 &= \frac{\tan 60 - \tan 45}{1 + \tan 60 \cdot \tan 45} \\ \tan 15 &= \frac{\sqrt{3} - 1}{1 + \sqrt{3} \cdot 1} \\ \tan 15 &= \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} \times \frac{\sqrt{3} - 1}{\sqrt{3} - 1} = \frac{3 + 1 - 2\sqrt{3}}{3 - 1} = \frac{4 - 2\sqrt{3}}{2} = (2 - \sqrt{3}) \end{aligned}$$

19. Jawab: A

$$\sin \alpha = \frac{4}{5} \text{ maka } \cos \alpha = \frac{3}{5} \text{ dan } \tan \alpha = \frac{4}{3} \text{ dan karena } \sin \beta = \frac{12}{13} \text{ maka } \cos \beta = \frac{5}{13} \text{ dan } \tan \beta = \frac{12}{5}.$$

Jadi

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \cdot \tan \beta} = \frac{\frac{4}{3} - \frac{12}{5}}{1 + \frac{4}{3} \cdot \frac{12}{5}} = \frac{\frac{(20 - 36)}{15}}{\frac{(15 + 48)}{15}} = -\frac{16}{63}$$

20. Jawab: D

$$\sin \alpha = \frac{4}{5} \text{ maka } \cos \alpha = \frac{3}{5} \text{ sehingga } \sin 2\alpha = 2 \cdot \sin \alpha \cdot \cos \alpha = 2 \cdot \frac{3}{5} \cdot \frac{4}{5} = \frac{24}{25}$$

21. Jawab: C

$$\sin \alpha = \frac{4}{5} \text{ maka } \cos \alpha = \frac{3}{5} \text{ sehingga } \cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

22. Jawab: D

$$\text{Dari soal no 20 dan 21 diperoleh bahwa } \tan \alpha = \frac{\sin 2\alpha}{\cos 2\alpha} = \frac{24}{7}$$

23. Jawab: D

$$\tan \alpha = \frac{1}{2} \text{ dan } \tan \beta = \frac{1}{3} \text{ maka } \tan 2\beta = \frac{2 \tan \beta}{1 - \tan^2 \beta} = \frac{2 \cdot \frac{1}{3}}{1 - (\frac{1}{3})^2} = \frac{\frac{2}{3}}{1 - \frac{1}{9}} = \frac{2}{3} \times \frac{9}{8} = \frac{3}{4} \text{ sehingga}$$

$$\tan(\alpha + 2\beta) = \frac{\tan \alpha + \tan 2\beta}{1 - \tan \alpha \cdot \tan 2\beta} = \frac{\frac{1}{2} + \frac{3}{4}}{1 - \frac{1}{2} \cdot \frac{3}{4}} = 2$$

24. Jawab: C

$$\tan \alpha = \frac{1}{2} \text{ dan } \tan \beta = \frac{1}{3} \text{ maka } \tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha} = \frac{2 \cdot \frac{1}{2}}{1 - (\frac{1}{2})^2} = \frac{1}{1 - \frac{1}{4}} = \frac{4}{3} \text{ sehingga } \tan(2\alpha - \beta) =$$

$$\frac{\tan 2\alpha - \tan \beta}{1 + \tan 2\alpha \cdot \tan \beta} = \frac{\frac{4}{3} - \frac{1}{3}}{1 + \frac{4}{3} \cdot \frac{1}{3}} = \frac{9}{13}$$

25. Jawab: C

$\cos 150 = p$ dengan menggunakan persamaan $\sin \frac{1}{2} \alpha = \pm \sqrt{\frac{1 - \cos \alpha}{2}}$, diperoleh

$$\sin 75 = \sin \left(\frac{1}{2} 150 \right) = \sqrt{\frac{1 - \cos 150}{2}} = \sqrt{\frac{1 - p}{2}}$$

26. Jawab: D

$\cos 150 = p$ dengan menggunakan persamaan $\cos \frac{1}{2} \alpha = \pm \sqrt{\frac{1 + \cos \alpha}{2}}$, diperoleh

$$\cos 75 = \cos \left(\frac{1}{2} 150 \right) = \sqrt{\frac{1 + \cos 150}{2}} = \sqrt{\frac{1 + p}{2}}$$

27. Jawab: C

Dari no 25 dan 26 diperoleh bahwa $\tan 75 = \frac{\sin 75}{\cos 75} = \sqrt{\frac{1-p}{1+p}}$

28. Jawab: E

$$4 \sin 45 \cos 15 = 2 \times (2 \sin 45 \cos 15) = 2 \times (\sin(45 + 15) + \sin(45 - 15))$$

$$4 \sin 45 \cos 15 = 2 \times (\sin 60 + \sin 30) = 2 \times \left(\frac{1}{2} \sqrt{3} + \frac{1}{2} \right) = \sqrt{3} + 1$$

29. Jawab: B

$$2 \cos a \sin 2b = \sin(a + 2b) - \sin(a - 2b)$$

30. Jawab: D

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos \alpha \sin \beta = \sin \alpha \cos \beta - \sin(\alpha - \beta)$$

$$\cos \alpha \sin \beta = \frac{1}{2} - \frac{1}{2} \sqrt{3}$$

sehingga diperoleh

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta = \frac{1}{2} + \frac{1}{2} - \frac{1}{2} \sqrt{3} = 1 - \frac{1}{2} \sqrt{3}$$